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Bottom Quark Mass Determination from low-n Sum Rules

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We study the uncertainties in the $\overline{\mathrm{MS}}$ bottom quark mass determination using relativistic sum rules to $\mathcal{O}(\alpha_S^2)$. We include charm mass effects and secondary $b\bar{b}$ production and treat the experimental continuum region more conservatively than previous analyses. The PDG treatment of the region between the resonances $\Upsilon(4S)$ and $\Upsilon(5S)$ is reconsidered. Our final result reads: $\bar{m}_b(\bar{m}_b) = (4.20 \pm 0.09)$ GeV.

For the sake of reliable measurements at the current B-factory experiments, a precise knowledge of the bottom quark mass m_b will be essential. In particular, the precision on the extraction of the Cabibbo–Kobayashi–Maskawa matrix elements V_{ib} from the data will depend on the uncertainty on the bottom mass. For example, an error of 60 MeV in m_b leads to a 3% uncertainty in V_{ub} from the semileptonic partial width $\Gamma(B \to X_u \ell\nu)$ [1].

In this talk we present the results of a detailed compilation of uncertainties in the $\overline{\mathrm{MS}}$ bottom quark mass [2]. Our analysis is more conservative than an earlier one in Ref. [3] and includes a number of effects that were previously neglected. Our method consists of determining the b mass by fitting the experimental moments of the $b\bar{b}$ cross section in e^+e^- annihilation to their correspondent theoretical expressions. The moments are defined as follows:

$$P_n = \int \frac{ds}{s^{n+1}} R_{b\bar{b}}(s), \tag{1}$$

where $R_{b\bar{b}} = \sigma(e^+e^- \to b\bar{b} + X)/\sigma(e^+e^- \to \mu^+\mu^-)$. The virtual Z contribution is strongly suppressed and neglected.

We use low-n ("relativistic") moments, i.e. $n \leq 4$. Relativistic moments exhibit the nice feature that they are dominated by scales of order m_b and that they can be computed in fixed-order perturbation theory. However, they have the disadvantage that they strongly depend on the badly known experimental continuum region. As for the b-mass definition, we adopt the $\overline{\mathrm{MS}}$ mass, since it is an appropriate definition for processes where b quarks are off-shell.

A compilation of the theoretical moments up to $\mathcal{O}(\alpha_S^2)$ was given in [3]. In our work [2] we also included the effects at $\mathcal{O}(\alpha_S^2)$ of the non-zero charm mass and of secondary $b\bar{b}$ production, with the $b\bar{b}$ pair coming from gluon radiation off light quarks. Referring, for semplicity, to $\bar{m}_b(\bar{m}_b)$ the theoretical moments have a simple form:

$$\begin{aligned} P_n \;=\; & \frac{1}{(4\bar{m}_b(\bar{m}_b))^n}\Bigg\{ f_n^0 + \left(\frac{\alpha_s(\mu)}{\pi}\right) f_n^{10} \\ & + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(f_n^{20}(r) - \frac{1}{4}\beta_0\, f_n^{10} \ln\left(\frac{\bar{m}_b^2(\bar{m}_b)}{\mu^2}\right)\right) \\ & + \frac{\langle \frac{\alpha_s}{\pi}\mathbf{G}^2\rangle}{(4\bar{m}_b(\bar{m}_b))^2}\left[g_n^0 + \left(\frac{\alpha_s(\mu)}{\pi}\right) g_n^{10}\right]\Bigg\}. \end{aligned} \tag{2}$$

In Eq. (2), $r = m_c/m_b$, μ is the renormalization scale and we have included the contribution from the dimension four gluon condensate [4]. The general expression of P_n for $\bar{m}_b(\mu)$ can be found in Refs. [2,3]. Results for the coefficients f_n are reported in Table 1. We point out that the contribution of secondary $b\bar{b}$ production and charm mass affects only f_n^{20}. Such effects turn out to be small, as can be seen comparing the values for f_n^{20} in Table 1 with the ones of Ref. [3]. Table 2 displays the impact of charm mass corrections in terms of $\Delta f_n = f_n^{20}(r) - f_n^{20}(0)$. The smallness of c-mass effects is however strongly related to the use of the $\overline{\mathrm{MS}}$ mass definition which we have adopted. In fact, if we had chosen the pole scheme for the bottom mass, the inclusion of the charm mass would have had a much stronger impact, as shown in Table 3. This can be understood from the fact that the finite charm mass represents an infrared cut-off in the loop integrations

Table 1
Coefficients of the theoretical moments in Eq. (2).

n	1	2	3	4
f_n^0	0.2667	0.1143	0.0677	0.0462
f_n^{10}	0.6387	0.2774	0.1298	0.0508
f_n^{11}	0.5333	0.4571	0.4063	0.3694
$f_n^{20}(0)$	0.9446	0.8113	0.5172	0.3052
f_n^{21}	0.8606	1.2700	1.1450	0.8682
f_n^{22}	0.0222	0.4762	0.8296	1.1240

Table 2
Corrections due to the non-zero charm quark mass in the $\overline{\mathrm{MS}}$ scheme.

r	0.1	0.2	0.3	0.4	0.5
Δf_1	-0.0021	-0.0078	-0.0164	-0.0266	-0.0382
Δf_2	-0.0028	-0.0091	-0.0187	-0.0302	-0.0430
Δf_3	-0.0024	-0.0101	-0.0204	-0.0330	-0.0466
Δf_4	-0.0030	-0.0109	-0.0219	-0.0348	-0.0491

and that the pole-mass definition is much more sensitive to infrared momenta. To evaluate the experimental moments, we consider the region of the resonances $\Upsilon(1S)-\Upsilon(6S)$ and the continuum. We compute the moments of a generic resonance k in the narrow width approximation, i.e.

$$(P_n)_k = \frac{9\pi\Gamma_k^{e^+e^-}}{\alpha(10\ \mathrm{GeV})m_k^{2n+1}}, \tag{3}$$

where $\Gamma_k^{e^+e^-}$ is the partial e^+e^- width for the k-th resonance. For the $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$ and $\Upsilon(6S)$ we use the averages for masses and widths quoted in the PDG [5]. For the region between the $\Upsilon(4S)$ and the $\Upsilon(5S)$, i.e. between 10.5 and 10.95 GeV, we do not use the PDG averages, which were based on results from CUSB [6] and CLEO [7] Collaborations. Both experiments observed an enhancement at about 10.7 GeV. While CUSB did not assign the enhancement to any resonance, CLEO fitted it to a "B^*" resonance with mass $m_{B^*} = 10.684 \pm 0.013$ GeV and e^+e^- width $\Gamma_{B^*}^{e^+e^-} = 0.20 \pm 0.11$ keV. The PDG averages, on the other hand, ignore the B^* results and, therefore, lead to a contribution to P_n that is smaller than the original CUSB and CLEO data. In our analysis we took the averages from the original CUSB and CLEO data, assuming the larger uncertainties from CLEO (See Ref. [2] for more details).

As far as the continuum is concerned, we subdivide it into three parts: 11.1-12.0 GeV (region 1), where possible data may come from CLEO; 12 GeV - M_Z (region 2) and above M_Z (region 3). There is no direct experimental data in the region above 11.1 GeV. Nevertheless the measurements of R_b by LEP I and LEP II agree with the perturbative QCD prediction within 1% at M_Z and 10% in the region between 133 and 207 GeV explored by LEP II. It is therefore not unreasonable to rely on perturbative QCD to estimate the contribution to the experimental moments above the $\Upsilon(6S)$. In the analysis of Ref. [3] the small theoretical errors in the continuum region were inherently taken as the experimental uncertainties. Since this leads to an implicit model-dependence, we adopt a more transparent treatment and take an assigned fraction of the theoretical prediction as the experimental uncertainty of the continuum. In this way the impact of the unknown experimental continuum contribution can be traced more easily. The experimental moments are quoted in Table 4. The uncertain-

Table 3
As in Table 2, but in the pole-mass scheme.

r	0.1	0.2	0.3	0.4	0.5
Δf_1	0.0809	0.1505	0.2113	0.2656	0.3145
Δf_2	0.0684	0.1267	0.1765	0.2203	0.2593
Δf_3	0.0608	0.1106	0.1531	0.1896	0.2221
Δf_4	0.0545	0.0988	0.1358	0.1676	0.1764

Table 4
Individual contributions to the experimental moments including uncertainties. In the continuum the displayed uncertainties are the theoretical ones only.

contribution	P_1 $\times 10^3$ GeV^2	P_2 $\times 10^5$ GeV^4	P_3 $\times 10^7$ GeV^6	P_4 $\times 10^9$ GeV^8
$\Upsilon(1S)$	0.766(29)	0.856(32)	0.956(36)	1.068(40)
$\Upsilon(2S)$	0.254(16)	0.252(16)	0.251(15)	0.250(15)
$\Upsilon(3S)$	0.211(29)	0.196(27)	0.183(26)	0.171(24)
$[\Upsilon(4S) - \Upsilon(5S)]$	0.251(95)	0.218(82)	0.190(72)	0.165(62)
Υ(6S)	0.048(11)	0.039(9)	0.032(7)	0.027(6)
11.1 GeV – 12.0 GeV	0.418(57)	0.314(44)	0.236(34)	0.178(27)
12.0 GeV – M_Z	2.467(26)	0.886(21)	0.414(13)	0.217(8)
$M_Z - \infty$	0.047(1)	0.000(0)	0.000(0)	0.000(0)

ties displayed for the three continuum regions are the ones from the theoretical uncertainties only.

For the determination of $\bar{m}_b(\bar{m}_b)$ and of the corresponding uncertainties, we use four methods: we fit single moments and get directly $\bar{m}_b(\bar{m}_b)$ (method 1); we determine $\bar{m}_b(\mu)$ from single-moment fits and subsequently $\bar{m}_b(\bar{m}_b)$ using renormalization group equations (method 2); we fit the ratio P_n/P_{n+1} and get $\bar{m}_b(\bar{m}_b)$ (method 3); we determine $\bar{m}_b(\mu)$ by fitting P_n/P_{n+1} and compute $\bar{m}_b(\bar{m}_b)$ using renormalization group equations (method 4). We employ four-loop renormalization group equations, vary the renormalization scale μ between 2.5 and 10 GeV and use $\alpha_s(M_Z) = 0.118 \pm 0.003$, $m_c = 1.3 \pm 0.2$ GeV, $\langle \frac{\alpha_s}{\pi} \mathbf{G}^2 \rangle = (0.024 \pm 0.024)$ GeV^4 as theoretical input. The results of our analysis are shown in Table 5. The central values for $\bar{m}_b(\bar{m}_b)$ according to the four methods agree within 15 MeV. For the errors coming from the experimental-continuum regions 2 and 3 we quote both, the one coming from the theory uncertainties shown in Table 4 and the error corresponding to a 10% variation of the theoretical prediction. The latter error scales roughly linearly, i.e. assuming a 5% (20%) fraction decreases (increases) the error by a factor of two. In order to get the combined errors, in the resonance region we treat half of the errors as uncorrelated (added linearly) and half of the errors as correlated (added quadratically). The errors in the continuum do not have any statistical correlation, hence we add them linearly. Moreover, we add linearly the errors coming from the resonance and from the continuum regions.

We note that the errors yielded by fits of the first two moments P_1 and P_2 are rather large. As for the results given by fits of the moment ratios, the fit of P_2/P_3 using method 3 yields a rather small error of about 50 MeV. However, this result holds only if the same value of μ is chosen for both P_2 and P_3; a larger error would instead be found using independent values of μ for the numerator and denominator of the ratio. Since we believe that P_3 can be calculated reliably using

Table 5
Central values and uncertainties for $\bar{m}_b(\bar{m}_b)$.

	Method 1 (2)			Method 3 (4)	
n	1	2	3	1	2
central	4210(4214)	4200(4205)	4197(4200)	4191(4195)	4191(4191)
$\Upsilon(1S)$	14 (13)	12 (12)	11 (11)	11 (11)	9 (9)
$\Upsilon(2S)$	7 (7)	6 (6)	5 (5)	4 (4)	3 (3)
$\Upsilon(3S)$	14(14)	10 (10)	8 (8)	7 (7)	3 (3)
$4S-5S$	45 (44)	32 (32)	22 (22)	18 (18)	4 (4)
$\Upsilon(6S)$	5 (5)	3 (3)	2 (2)	2 (2)	0 (0)
combined	67 (67)	50 (50)	38 (38)	33 (33)	15 (15)
[region 1]$_{\rm th}$	$27_{\rm th}$ ($26_{\rm th}$)	$17_{\rm th}$ ($17_{\rm th}$)	$11_{\rm th}$ ($11_{\rm th}$)	$7_{\rm th}$ ($7_{\rm th}$)	$2_{\rm th}$ ($2_{\rm th}$)
[region 2]$_{\rm th}$	$12_{\rm th}$($12_{\rm th}$)	$8_{\rm th}$ ($8_{\rm th}$)	$4_{\rm th}$ ($4_{\rm th}$)	$4_{\rm th}$ ($4_{\rm th}$)	$4_{\rm th}$ ($4_{\rm th}$)
[region 2]$_{10\%}$	115 (114)	33 (33)	13 (13)	49 (49)	29 (29)
[region 3]$_{\rm th}$	$1_{\rm th}$ ($1_{\rm th}$)	$0_{\rm th}$ ($0_{\rm th}$)	$0_{\rm th}$ ($0_{\rm th}$)	$1_{\rm th}$ ($1_{\rm th}$)	$0_{\rm th}$ ($0_{\rm th}$)
[region 3]$_{10\%}$	2 (2)	0 (0)	0 (0)	2 (2)	0 (0)
δm_c	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
$\delta\alpha_s(M_Z)$	17 (18)	10 (11)	6 (6)	3 (3)	2 (2)
$\delta\mu$	23 (5)	16 (14)	11 (27)	15 (27)	3 (50)
combined	184 (166)	77 (75)	41 (57)	76 (88)	37 (85)
total	251 (233)	127 (125)	79 (95)	110 (121)	51 (99)

both methods 1 and 2, we adopt the error on P_3 as our final estimate of the uncertainty in the $\overline{\rm MS}$ bottom mass determination. Rounding to units of 10 MeV, we obtain:

$$\bar{m}_b(\bar{m}_b) = (4.20 \pm 0.09)\ \text{GeV}, \tag{4}$$

assuming a 10% error for the experimental continuum regions 2 and 3. Within the error range, our result is in agreement with the estimate of [3]. Our error is nonetheless larger than the 50 MeV of Ref. [3], which is due to the different treatment of the resonance region and to the more conservative choice for the experimental error in the continuum region.

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